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ABSTRACT

This document examines Colorado's size factor, which is intended to compensate for differences in per-pupil costs that are attributable to economies of scale; the formula provides additional money based on each district's enrollment. The report is organized by the following three areas: (1) background on Colorado's size factor, including a description of the formula, a summary of changes in the factor over time, and a comparison with similar factors in other states; (2) a review of the actual factors and the funding that is driven by the factors in Fiscal Year 1999-00; and (3) a review of research and literature on economies of scale in general. The report focuses on changes to the size factor under the School Finance Act and offers a synopsis of 10 states that incorporated a size factor in their school-finance formulas. It shows how measures of economies of scale were taken and what the impact of the size factor on per-pupil funding would be. The findings reveal that in Fiscal Year 1998-99, the size factor contributed $121 million toward total programs, or nearly 4 percent of the total funding provided through Colorado's School Finance Act. Districts with the fewest pupils received the largest size factor. On average, the size factor added $186 to per-pupil funding. Three appendices show how to calculate size factors and feature other information. (RJM)

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Legislative Council Staff Report on

School District Size Factors

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Report to the

COLORADO

GENERAL ASSEMBLY

December 1998

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School District Size Factors

Report to the
Colorado General Assembly

Colorado Legislative Council
Research Publication No. 451
January 1999

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January 1999

To Members of the Sixty-second General Assembly:

Submitted herewith is the study of School District Size Factors. The study is required pursuant to Section 22-54-104 (5) (b) (I.3) (B), C.R.S.

Respectfully submitted,

Charles S. Brown
Director

CB/CW/cs

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EXECUTIVE SUMMARY

Study Charge

Section 22-54-104 (5) (b) (I.3) (B), C.R.S., directs the Legislative Council to conduct a study of the size factors established for FY 1999-2000 and make a report of its findings to the General Assembly no later than January 15, 1999.

Staff Activities

This study examines Colorado's size factor, as well as the concept of educational economies of scale. Our work was organized along the following three areas:

- background on Colorado's size factor, including a description of the formula, a summary of changes in the factor over time, and a comparison with similar factors in other states,
- a review of the actual factors and the funding that is driven by the factors in FY 1999-00, and
- a review of research and literature on economies of scale in general, and especially in the field of education.

Study Findings

Colorado's school finance act includes factors intended to compensate districts for cost pressures that are beyond their control. The size factor is intended to compensate for differences in per pupil cost which are attributable to economies of scale and provides additional money based on each district's enrollment. In FY 1998-99, the size factor contributed $121 million toward total program, or nearly four percent of the total funding provided through Colorado's school finance act.

Ten states incorporate a factor similar to Colorado's size factor in their respective school finance formulas, but only Colorado provides additional funding to all districts in the state. Beginning in FY 1999-00, the factor affects districts as follows:

- districts with the fewest pupils receive the largest size factor (up to a maximum possible factor of 2.5884),
- medium-sized districts receive a size factor of at least 1.0120, and
- large districts receive a factor that increases up to 1.0342 for districts with 32,193 or more pupils.

Because the factor provides more additional funding to small and large districts than to medium-sized districts, it is often referred to as the "J" curve.

The concept of economies of scale implies that it costs more per pupil to educate pupils in small school districts than in large school districts. Several studies have documented the relatively high per pupil costs in small districts, especially those caused by pupil-teacher ratios, fixed overhead, or purchasing. However, not all researchers agree that bigger districts are more cost-efficient; some also support the idea that *di*seconomies of scale may cause per pupil costs to increase again as the size of the school district increases beyond an optimal point.

The study of economies of scale usually involves analyzing the per pupil cost of educating students, or at least variables that approximate differences in per pupil costs attributable to economies of scale. Colorado's size factor is based on expenditure data modified to control for a number of factors and eliminate some potential biases. The "J" curve uses a methodology called LOWESS to compare a size variable (measured by pupil enrollment) and a per pupil cost proxy (modified per pupil expenditures).

The size factor directly increases a district's per pupil funding. For example, a size factor of 2.0000 effectively doubles a district's per pupil funding. On average, the size factor adds $186 to per pupil funding, although this figure varies greatly among individual districts. The value of the size factor to any particular district can be calculated by the following formula:

Per Pupil Revenue from Size Factor = (district size factor - 1) x (statewide base)

Based on the most recent FY 1999-00 estimates available, the Legislative Council Staff projects the following:

- the state's 155 smallest districts (less than 5,650 pupils each) will enroll about 25 percent of the state's students, receive size factors ranging from 2.5884 to 1.0120, and consume 64 percent of the money distributed through the size factor ($81 million);

- the 13 medium-sized districts (between 5,650 and 25,546 pupils each) will enroll about 25 percent of all students, receive the minimum size factor of 1.0120, and consume about six percent of the size factor money ($8 million); and

- the 8 largest districts (over 25,546 pupils each) will enroll about 50 percent of all students, receive size factors between 1.0120 and 1.0342, and consume the remaining 30 percent of size factor moneys ($38 million).

On average, districts in these three tiers receive $481, $45, and $112, respectively, per student from the size factor. The minimum dollar increase provided through the size factor is $45, while the maximum is $5,279.

I. BACKGROUND

This chapter provides an introduction to Colorado's size factor as well as a review of how the size factor was established, how it has changed, and how it compares with similar factors in other states.

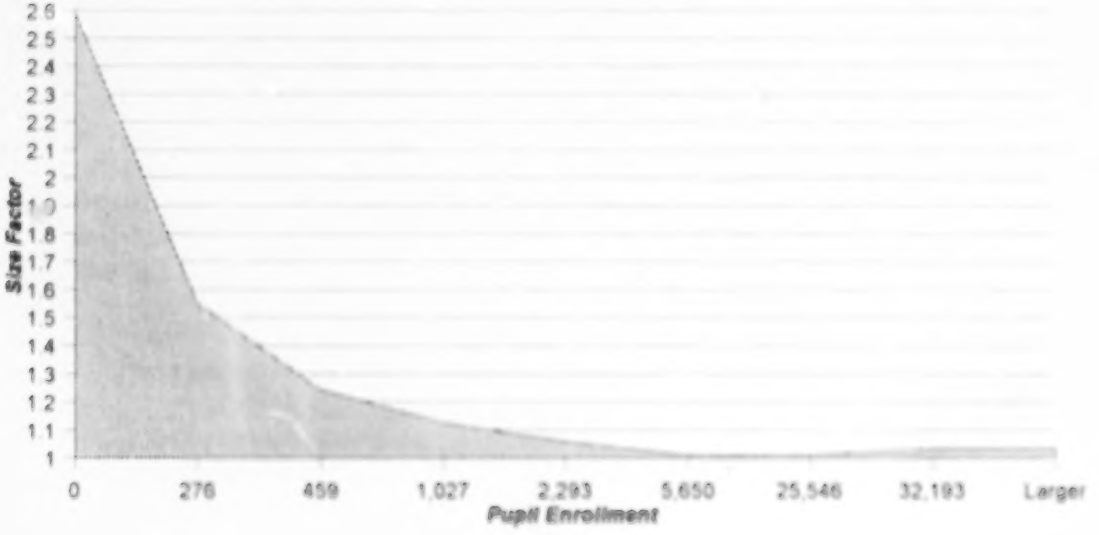
Introduction to Colorado's Size Factor

The size factor (or "J" curve) in Colorado's school finance act is a key component of funding for public schools. In FY 1998-99, the size factor accounts for roughly $121 million, or nearly four percent of the act's total funding. The factor compensates districts based on the theory that per pupil costs are subject to economies of scale.

What is the "J" curve? The term "J" curve is used to describe the shape resulting from school district size factors, when graphed by enrollment. Chart 1, below, depicts the size factor formula established for FY 1999-00. Please note the chart is not drawn to scale.

CHART 1: The Size Factor ("J" Curve)

NOT DRAWN TO SCALE



The Legislative Council Staff estimates that one quarter of the students funded under Colorado's school finance act will be enrolled in the state's 155 smallest districts in FY 1999-00. These districts enroll less than 5,650 pupils each and receive size factors ranging from 1.0120 to 2.5884. Another one quarter of students are expected to enroll in the 13 districts with between 5,650 and 25,546 pupils. Districts in this enrollment range receive the minimum size factor of 1.0120 in

FY 1999-00. The remaining half of the students are expected to enroll in the state's eight largest districts. These eight districts are expected to enroll more than 25,546 pupils each and will receive size factors between 1.0120 and 1.0342 in FY 1999-00. Over half of the students in large districts are enrolled in the three largest districts in the state and will receive a size factor of 1.0342.

How does the "J" curve affect per pupil funding? The size factor directly increases a district's per pupil funding. For example, a size factor of 2.0000 effectively doubles a district's per pupil funding. A size factor of 1.5000 increases per pupil funding by about 50 percent. A size factor of 1.0120 increases per pupil funding by about 1.2 percent. A size factor of 1.0342 increases per pupil funding by about 3.42 percent. On average, the size factor adds $181 to per pupil funding in FY 1998-99, although this figure varies greatly among individual districts. The effect of the size factor on district funding is discussed in greater detail in Chapter II of this study.

The Origin of Colorado's Size Factor

The size factor became law with the passage of House Bill 94-1001, which created the Public School Finance Act of 1994. It was recommended by the 1993 Interim Committee on School Finance to compensate school districts for economies-of-scale cost pressures that are beyond the district's control. The idea that it is appropriate for the state to compensate districts for the cost pressures created by the number of pupils enrolled was also recommended by an advisory committee comprised of school finance experts that met in 1992 and 1993 to discuss school finance in Colorado.

The factor approximates differences in per pupil cost. Through the size factor, the General Assembly has established a policy of compensating districts for economies-of-scale cost pressures through the state's school finance formula. Implementing the policy, however, requires measuring exactly how economies of scale affects the per pupil costs faced by Colorado school districts and consistent and reliable cost data are not available. Therefore, the factor was based on a proxy using the best data available to approximate per pupil costs. The proxy was developed using actual district expenditures modified to control for a number of factors and eliminate some potential biases. It was then compared on a graph with district enrollment and a line was plotted to determine the central tendency of districts. This line is replicated in the size factor's statutory formula.[1]

Changes to the Size Factor Under the School Finance Act

As previously mentioned, the size factor is calculated under a formula in the school finance act. Thus, a change in the formula requires an amendment to the law. Since enactment of the size factor formula in 1994, the General Assembly has considered several modifications, although the substance of the statutory formula had remained essentially unchanged between 1994 and 1998. Prior to FY 1998-99, the factor provided funding to only the smallest and the largest districts, beginning with FY 1998-99, the formula is modified to provide additional funding to *all* districts.

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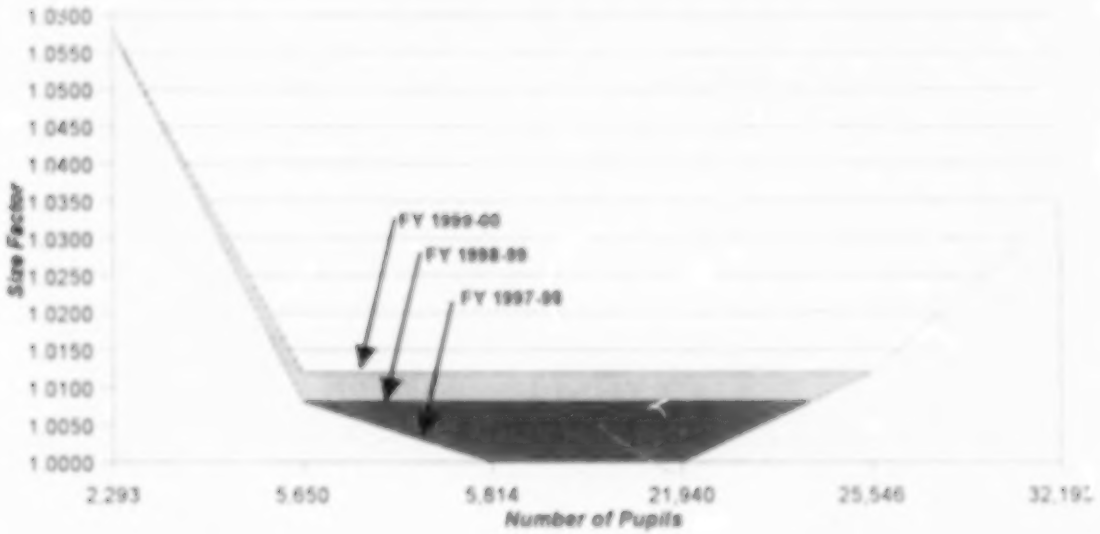
1. The line plotted against the data incorporated a method of weighting data called LOWESS. See Chapter III for further explanation of how the LOWESS methodology was applied in Colorado and Appendix 2 for a detailed explanation of LOWESS.

The first change occurred before the bill was even adopted. Relatively early in the debate, the legislature altered the interim committee's original formula so that a range of districts (with enrollments between 5,814 and 21,940) received a size adjustment factor of 1.0000, where before only one district (with enrollment of 17,659) received a size adjustment factor of 1.0000. Some legislators argued that there were little noticeable differences in the economies-of-scale cost pressures experienced by districts with between 5,814 and 21,940 pupils, others commented that the original formula adversely affected one district disproportionately. The net effect of the formula's alteration was to decrease the amount of funding received by all districts under the size factor. The legislature considered other changes as well, including at least two bills (outside the school finance bill) that would have guaranteed a minimum size factor.[2]

A minimum size factor is established beginning in FY 1998-99. The most significant change to the size factor occurred in 1998. With the passage of House Bill 98-1234, the General Assembly modified the size factor formula to phase-in additional funding for districts with between 2,293 and 25,546. For FY 1998-99, the school finance act guarantees that no district will have a size factor less than 1.0081. For FY 1999-00 and thereafter, the law provides a minimum size factor of 1.0120. Chart 2, below, highlights these recent changes to the size factor formula.

CHART 2: Recent Changes to the Size Factor

NOT DRAWN TO SCALE



Other changes affect reorganized districts and small districts with charter schools. Aside from the issue of a minimum size factor, there have been two other changes to the formula. First, the General Assembly modified the formula to account for school district reorganizations. Under the revised formula, when a reorganization results in a lower size factor, and less funding per pupil, the lower size factor is phased in over six years. Conversely, when a reorganization results in a higher

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2. House Bill 97-1135 would have established a minimum size factor of 1.0342. House Bill 95-1051 would have established a minimum size factor of 1.0160.

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size factor, and more funding per pupil, the district or districts involved in the reorganization receive the lower size factor of the original district. Thus, the act lessens the negative fiscal impact of reorganization while prohibiting a district from taking advantage of a higher size factor following a reorganization.

Second, the General Assembly provided additional money to minimize the effect that charter schools may have on the size factor of small school districts. Beginning in FY 1998-99, the size factor for districts with less than 500 pupils is calculated using the district's enrollment, minus 65 percent of the pupils enrolled in charter schools. This change provides a higher size factor, and higher per pupil funding, to small districts with charter schools than such districts would otherwise receive.

Ten States Incorporate a Size Factor in their School Finance Formulas

We identified ten states that use district or school enrollment as a basis for providing additional funding in their school finance formulas. These states include Alaska, Arizona, California, Colorado, Florida, Kansas, Nebraska, New Mexico, Oklahoma, and Texas.[3] The Kansas formula is of particular interest because it was studied extensively by the legislature in 1994 while the supreme court in that state considered the constitutionality of the formula. The legislature's study confirmed some basic economy-of-scale ideas, including the following:

- it costs more per pupil to offer an equivalent educational program in smaller enrollment districts than it does in larger enrollment districts,
- pupil/teacher ratios appear to be the greatest contributor to high per pupil costs, and
- it can be difficult to identify variables related to cost which do not reflect historical expenditures.

All ten state formulas provide the most money to the smallest districts, on a per pupil basis, thus incorporating the idea that smaller school districts cannot take advantage of the economies of scale available to larger school districts. Colorado's formula provides the greatest per pupil increase for very small districts, followed by Alaska and Kansas. Colorado's size factor formula is the broadest, applying to *all* districts in the state. Alaska provides aid to all districts in the state except the state's largest. Meanwhile, Oklahoma provides size adjustment funding to the smallest enrollment range of districts — only those with less than 529 pupils.

Seven of the ten states have size adjustment formulas which provide additional funding only for small districts or schools (i.e., those with less than a specific enrollment level). The other three states — Colorado, Nebraska, and New Mexico — have formulas which mirror a "J" curve, such that the smallest enrollment districts receive the largest size adjustment but the largest districts also receive additional funding. Of these states, New Mexico provides the most per pupil to large districts.

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3. Other states may offer size-adjustment programs but differences in school finance formulas make comparisons to Colorado difficult.

Most of the ten state formulas measure size using the number of pupils in the district. However, there are two notable exceptions. New Mexico's size adjustment formula also considers the number of pupils in each school, and Alaska's size adjustment formula uses community populations which represent only a portion of the district's population.

It is worth noting that some states also provide funding to account for geographic isolation as a barrier to economies of scale. For example, Colorado allocates money to small, isolated schools, Texas provides additional funding to districts larger than 300 square miles, and Oklahoma allows districts to use either the size adjustment formula or an isolation formula, whichever provides greater assistance.

II. ECONOMIES OF SCALE LITERATURE AND RESEARCH

This chapter examines literature and research on economies of scale in education. First, we define economies of scale and introduce the firm (or the school district) as the primary basis for measuring economies of scale. Second, we review research and literature related to economies of scale and, in particular, research that examines how the number of pupils in a school district affects the per pupil cost of educating students. Third, we discuss issues related to measuring economies of scale such as the availability of data and alternatives for analyzing data, giving particular emphasis on the LOWESS method used to develop the size factor in Colorado. A portion of this chapter builds upon previous research published by the Legislative Council Staff.[4]

Economies of Scale, Defined

Economic theory holds that the cost to produce an individual item decreases with an increase in the number of items produced. This phenomenon is often called "economies of scale" because it implies that a firm can achieve economic savings by increasing its scale of production.

In education, the theory of economies of scale implies that it costs more per pupil to educate pupils in small school districts than in large school districts. One way to illustrate this is to compare fixed costs spread over a district's enrollment. For example, all districts must employ a superintendent, whose salary can be converted into a per pupil cost. If the salary for a superintendent is $100,000, this translates into costs of $10 per pupil in a district with 10,000 pupils and $1,000 per pupil in a district with 100 pupils.

Economic theory also supports the idea that reduced costs of providing education may only be present for larger school districts up to an optimal size. In other words, *dis*economies of scale may cause per pupil costs to increase again as the size of the school district increases beyond an optimal point.

To acknowledge that economies of scale exist in providing education services in Colorado, it logically follows that they be considered in educational funding. The General Assembly compensates school districts for enrollment-based cost pressures through a size factor in the state's school finance act.

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4. This earlier research is contained in three published studies: *Legislative Council Staff Report on the School District Setting Category Study*, Colorado Legislative Council Research Publication No. 376, March 1993; *Legislative Council Staff Report on the Senate Bill 93-87 Setting Category Study*, Colorado Legislative Council Research Publication No. 378, August 1993; and *School Finance Study*, Colorado Legislative Council Research Publication No. 398, January 1995.

A Review of Economies of Scale Research and Literature

Most economy of scale research is based on production by a firm. The general consensus among researchers is that larger firms can capitalize on the financial benefits of mass production such as more efficient use of resources and specialization of labor. For example, a large firm might have more capital resources available to automate the production of goods, whereas a small firm could not afford such efficiencies. Similarly, a large firm purchasing large quantities of raw materials might pay less per unit than a small firm purchasing smaller amounts of raw materials.

Some research also points to inefficiencies for very large production firms, implying that there is something like an "optimal" size operation. As the size of a firm increases up to a certain point, average unit costs decrease and then level off. Above this size, average unit costs may actually increase where the production curve of the firm becomes "U-shaped."[5] One possible explanation for this diseconomies-of-scale phenomenon is that the infrastructure upon which the firm relies is inadequate to handle the volume of goods produced.

Economies of scale in education. Research and literature also point to economies of scale in education, implying that students can be taught most efficiently, or at a lower per pupil cost, in districts of a certain size. Nearly all of the relevant research tends to support the idea that small school districts face relatively high per pupil costs. These relatively high per pupil costs in small districts are often created by the required minimum level of education inputs. Using the example from earlier in this chapter, the district with 10,000 pupils might have 800 high school seniors and 32 high school teachers, producing a pupil-teacher ratio of 25:1. Keeping proportions the same, the district with 100 pupils would have 8 high school seniors and one high school teacher, producing a pupil-teacher ratio of 8:1. If the salary for a teacher is $25,000, the cost in the district with 10,000 pupils is $1,000 per pupil while the cost in the district with 100 pupils is $3,125 per pupil. These costs are illustrated in Table 1.

TABLE 1: Comparison of Per Pupil Costs in Large and Small Districts

	District A	District B
Enrollment	10,000 pupils	100 pupils
Estimated High School Seniors (1/12th of total)	800	8
Number of High School Teachers	32	1
Pupil-Teacher Ratio	25:1	8:1
Teacher Salary	$25,000	$25,000
Per Pupil Cost	$1,000	$3,125

5. The "U-shaped" production curve is replicated, to a limited extent in Colorado's size factor, which is often called the "J" curve. The concept is the same, however, in that they both indicate higher unit costs for organizational units that grow beyond an optimal size.

Besides educational inputs, researchers have found that economies of scale can affect the per pupil cost of administration, building maintenance, support programs, and purchasing of equipment, supplies, and other materials. One study found economies of scale to exist in all nine of the states included in the research.

To lower the cost of education, some small districts have turned to the following:

- collective purchasing of equipment, supplies, and other materials;
- technology improvements so students can access programs offered at other districts or higher education institutions; and
- cooperative arrangements to jointly provide expensive educational services, such as those for severely handicapped students.

As a last resort, many states have pursued school district consolidation, although these efforts have subsided. School district consolidation often means closing smaller schools and combining (or eliminating) programs with a high per pupil cost. There are several barriers, however, that may affect the ability of a school district to achieve economies of scale: communities may object to the loss of control over local schools; citizens may object to closing schools that contribute to a community's identity; and it may cost more to transport students in the new district. In addition, some research also confirms a U-shaped cost curve with increasing per pupil costs for districts that grow beyond an optimal size. The nine-state study noted above found evidence to support the theory that diseconomies of scale arise when size exceeds the optimum, although the optimum size varied in each state. Another study found evidence that average costs decrease at a decreasing rate as enrollment increases and that instructional unit costs begin to rise again as institutions become very large. These and other studies have found that relatively large institutions had higher unit costs than mid-sized institutions, confirming a U-shaped cost curve.

Measuring Economies of Scale

While researchers may agree that economies of scale exist, they often disagree on the appropriate means for measuring these economies of scale. In particular, there may be disagreement over both the type of data and the type of analysis. These and other important issues are described in detail below.

Data availability. The most widely accepted method of measuring economies of scale involves analyzing the cost of inputs such as raw materials and the means of production. When data are available, economies of scale can be measured using regression or other statistical techniques. For service industries like education, however, consistent and reliable data are often difficult to obtain.

Without accurate, consistent, and reliable cost data, some statistical analyses rely on other variables as proxies. Proxies are variables used in econometric modeling that are thought to be highly correlated to the unavailable data. In education, per pupil expenditures are often used as a proxy for per pupil cost, sometimes in conjunction with pupil-teacher ratios and average teacher salaries. When measuring the effect of economies of scale in education, a proxy may also include expenditures for

certain fixed inputs which are necessary for a district's operation regardless of the number of students enrolled in the district. These fixed inputs include items such as building maintenance, overhead, and some administrative and support services.

In Colorado, data are available on many of the variables that affect the cost of education, such as student characteristics, average salaries, pupil-teacher ratios, and overhead expenditures. Data are not available, however, on the actual cost to educate students or how much each variable contributes to this cost. Thus, to estimate costs in Colorado a proxy was developed using actual district expenditures modified to reflect per pupil costs. The expenditures were modified to control for a number of factors and eliminate some potential biases. After modification, the figures represented the best data available to approximate per pupil costs.

Data analysis. The debate over how to measure economies of scale is not limited to the data, it also encompasses the method for analyzing data. Several statistical alternatives are available, including regression analysis and a method of weighting data called LOWESS. Regression analysis is often applied to this type of research and confirmed a relationship between size and per pupil costs. Limitations on the data, however, caused the regression models we examined during development of the size factor to indicate a static relationship between size and per pupil costs over all enrollment levels. Thus, to measure whether the relationship between size and per pupil costs changes over different enrollment levels using regression modeling would have required segregating the data into categories, reducing the data elements available for analysis, and possibly reducing the quality of the analysis. LOWESS, on the other hand, seems to better accommodate the possibility of a dynamic relationship between size and per pupil costs using all the available data.

Colorado's size factor applies the LOWESS methodology to a size variable (measured by pupil enrollment) and a per pupil cost proxy (modified per pupil expenditures). To develop the proxy, the expenditure data were modified to control for a number of factors and eliminate some potential biases. Specifically, the data were subject to the following four modifications:

- first, 1991 expenditures were inflated to reflect then-current FY 1993-94 school finance moneys, after phase-in of the 1988 school finance act;
- second, total expenditures were divided by pupil enrollment to determine an average per pupil amount;
- third, the per pupil amount was divided by each district's cost-of-living factor to account for regional differences in the costs of housing, goods, and services, and
- fourth, the per pupil amount was reduced by $313 to account for the fact that all districts were required by law to devote at least that much per pupil for instructional supplies and materials, capital reserve, and insurance reserve.

The LOWESS line resulting from the graph of modified per pupil expenditures and pupil enrollment revealed that the relationship between size and costs was dynamic and curvilinear. It also revealed specific enrollment levels where the relationship between size and costs seemed to change. The per pupil expenditure levels at these enrollment levels were converted into factors (dividing by

the lowest so that the minimum size factor was 1.00) and the mathematical slope of the line between the factor at each enrollment level was calculated. This line is replicated in the size factor's statutory formula which can be found in Appendix 1.

Additional issues. At the same time researchers generally support the idea that economies of scale exist, and that states should compensate districts for the resulting cost pressures, they also point out the difficulty of considering enrollment as a cost factor in isolation. Even at a theoretically "efficient" enrollment level, districts may face different cost pressures based on one or more of the following:

- available facilities and capacity utilization;
- cost of pupil transportation;
- community expectations; and
- geographic sparsity.

The trade-off between costs and benefits, and the decisions made by local school boards in this trade-off, add to the difficulty of measuring economies of scale. Also adding to the difficulty, according to many researchers, is the problem of how to measure the consistency and quality of educational outputs. A firm can measure the quality of each unit produced and impose quality control systems to ensure consistency, but quality is much more difficult to measure in education, where students may respond differently to different educational settings.

The most common measure of educational output is standardized test scores, although output is also sometimes measured by the number of graduates or their grade point average. Many authors note that the output of the educational system goes far beyond measurable test scores, and that a proper economy-of-scale model would need to account for these outputs as well. However, just as with the cost of educational inputs, good surrogates for output measures are usually difficult to obtain. By not including a measure of output, Colorado's size factor, by implication, assumes that all districts are providing the same quality, or at least an acceptable quality, of education.

III. SIZE FACTORS AND FUNDING FOR FY 1999-00

The size factor provides a direct, measurable, and sometimes large share of a district's per pupil funding. This chapter illustrates exactly how the size factor affects Colorado school districts. First, it illustrates how the factor is calculated. Second, it discusses the impact of the size factor on per pupil funding. Third, it provides an estimate of the FY 1999-00 factor for each district and amount of money driven by the factor in total and on a per pupil basis.

Calculating the Size Factor

Each district's size factor is calculated under a formula contained in law which uses the count of pupils. The complete formula for calculating school district size factors is in Appendix 1. For a district with 150 pupils, the size factor is calculated below:

statutory formula:	=	*1.5502 + (0.00376159 x the difference between the funded pupil count and 276)*
size factor for a district with 150 pupils	=	*1.5502 + (0.00376159 x (276-150))*
	=	*1.5502 + (0.00376159 x 126)*
	=	*1.5502 + 0.4740*
	=	*2.0242*

Impact of the Size Factor on Per Pupil Funding

As noted above, the size factor can have a large impact on a district's per pupil funding. On average, the size factor adds $186 to per pupil funding, although this figure varies greatly among individual districts. The value of the size factor to any particular district can be calculated by the following formula:

Per Pupil Revenue from Size Factor = (district size factor - 1) x (statewide base)[6]

Smaller districts tend to have a larger size factor and higher per pupil funding. For example, the factor for a district with 150 pupils (2.0242) provides 19 times more of an increase in per pupil funding than the factor for a district with 2,500 pupils (1.0550) and 85 times more than the factor for a district with 15,000 pupils (1.0120). The impact levels out, however, so that the factor for the district with 2,500 pupils provides five times more of an increase in per pupil funding than the factor for the district with 15,000 pupils and double the increase for a district with 35,000 pupils (1.0342). The factor for the district with 35,000 pupils provides an increase in per pupil funding three times higher than the factor for the district with 15,000 pupils. Table 2, on the next page, illustrates these figures.

6. The statewide base is set annually by the General Assembly. For FY 1998-99, the statewide base is $3,783.

TABLE 2: A Sample of Size Factors and Size Factor Funding FY 1999-00

District Funded Pupil Count	FY 1999-00 Size Factor	Per Pupil Funding Increase from Size Factor *(base X size factor)*	Total Funding Increase from Size Factor *(base X size factor X pupils)*
150	2.0242	$3,875	$581,182
2500	1.0550	$208	$520,162
15,000	1.0120	$45	$680,940
35,000	1.0342	$129	$4,528,251

Note: numbers may not sum due to rounding.

The size factor also creates a compounding effect on per pupil revenue, because it is applied to a figure that has already been adjusted for cost of living. For example, in FY 1998-99, the formula is expected to provide roughly $475 million from the cost-of-living factor, $121 million from the size factor, and $18 million from the compounding of the two factors.

Estimated FY 1999-00 Enrollment and Size Factors

Table 3, on the next few pages, illustrates the estimated enrollment, the calculated size factor, and the estimated revenue produced by the size factor for each school district in Colorado.

Based on the most recent FY 1999-00 estimates available, the 155 smallest districts — those with enrollments of 5,650 or less — will consume 64 percent of the money distributed through the size factor, while accounting for about one quarter of the students funded under the school finance act. Eighty-one million dollars of the $127 million allocated through the size factor will be distributed to these districts. The next tier of districts — those with enrollments between 5,650 and 25,546 — will receive just over six percent, or about $8 million, of the size factor money. As mentioned previously, the 13 districts in this tier enroll another one quarter of the students in Colorado. Finally, the last tier of districts — those with enrollments greater than 25,546 — will receive the remaining 30 percent of size factor moneys. This 30 percent translates into about $38 million for the half of the state's students enrolled in these eight districts. On average, districts in these three tiers receive $481, $45, and $112, respectively, per student from the size factor.

While the size factor tends to provide most districts with roughly $500,000 in total, the amount received per pupil from the factor varies widely. The minimum dollar increase provided through the size factor is $45, while the maximum is $5,279. When examined in terms of total per pupil funding from the school finance act, the money received under the size factor ranges from a low of nine-tenths of one percent to a high of 54 percent.

TABLE 3. Estimated Enrollment, Size Factors, and Size Factor Funding, FY 1999-00

	COUNTY	DISTRICT	EST. OCT 99 FUNDED PUPIL COUNT	EST. FY99-00 SIZE FACTOR	TOTAL REVENUE FROM SIZE FACTOR *	PER PUPIL REVENUE FROM SIZE FACTOR *
1	ADAMS	MAPLETON	4,711.6	1.0248	442,035	94
2	ADAMS	NORTHGLENN	27,710.0	1.0192	2,012,677	73
3	ADAMS	COMMERCE CITY	5,696.0	1.0120	258,576	45
4	ADAMS	BRIGHTON	4,839.0	1.0231	422,867	87
5	ADAMS	BENNETT	967.8	1.1382	505,976	523
6	ADAMS	STRASBURG	599.0	1.2142	485,381	810
7	ADAMS	WESTMINSTER	11,034.0	1.0120	500,899	45
8	ALAMOSA	ALAMOSA	2,415.8	1.0561	512,696	212
9	ALAMOSA	SANGRE DECRISTO	316.0	1.4831	577,511	1,828
10	ARAPAHOE	ENGLEWOOD	4,377.0	1.0294	486,811	111
11	ARAPAHOE	SHERIDAN	2,099.0	1.0683	542,337	258
12	ARAPAHOE	CHERRY CREEK	40,237.0	1.0342	5,205,807	129
13	ARAPAHOE	LITTLETON	15,797.5	1.0120	717,143	45
14	ARAPAHOE	DEER TRAIL	178.3	1.9177	618,997	3,472
15	ARAPAHOE	AURORA	27,833.0	1.0196	2,063,728	74
16	ARAPAHOE	BYERS	453.0	1.2531	433,737	957
17	ARCHULETA	ARCHULETA	1,594.5	1.0954	575,452	361
18	BACA	WALSH	233.5	1.7101	627,253	2,686
19	BACA	PRITCHETT	95.0	2.2310	442,403	4,657
20	BACA	SPRINGFIELD	359.3	1.4104	557,829	1,553
21	BACA	VILAS	78.1	2.2946	382,493	4,897
22	BACA	CAMPO	81.8	2.2807	396,312	4,845
23	BENT	LAS ANIMAS	728.8	1.1874	516,671	709
24	BENT	MCCLAVE	261.1	1.6062	598,769	2,293
25	BOULDER	ST VRAIN	17,638.0	1.0120	800,695	45
26	BOULDER	BOULDER	26,371.0	1.0148	1,476,470	56
27	CHAFFEE	BUENA VISTA	888.90	1.1544	519,202	584
28	CHAFFEE	SALIDA	1,286.1	1.1120	544,915	424
29	CHEYENNE	KIT CARSON	133.5	2.0862	548,564	4,109
30	CHEYENNE	CHEYENNE R-5	327.4	1.4639	574,565	1,755
31	CLEAR CREEK	CLEAR CREEK	1,367.5	1.1077	557,159	407
32	CONEJOS	NORTH CONEJOS	1,181.0	1.1177	525,851	445
33	CONEJOS	SANFORD	351.0	1.4243	563,400	1,605
34	CONEJOS	SOUTH CONEJOS	443.6	1.2689	451,252	1,017
35	COSTILLA	CENTENNIAL	355.1	1.4174	560,711	1,579
36	COSTILLA	SIERRA GRANDE	334.5	1.4520	571,967	1,710
37	CROWLEY	CROWLEY	619.0	1.2100	491,752	794
38	CUSTER	WESTCLIFFE	392.0	1.3555	527,184	1,345
39	DELTA	DELTA	4,554.0	1.0269	463,427	102
40	DENVER	DENVER	65,470.5	1.0342	8,470,482	129
41	DOLORES	DOLORES	330.9	1.4580	573,322	1,733
42	DOUGLAS	DOUGLAS	30,979.0	1.0301	3,527,526	114
43	EAGLE	EAGLE	4,293.0	1.0305	495,333	115
44	ELBERT	ELIZABETH	2,552.0	1.0543	524,224	205
45	ELBERT	KIOWA	357.0	1.4142	559,390	1,567
46	ELBERT	BIG SANDY	374.5	1.3848	545,159	1,456
47	ELBERT	ELBERT	260.5	1.6085	599,659	2,302

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	COUNTY	DISTRICT	EST. OCT 99 FUNDED PUPIL COUNT	EST. FY99-00 SIZE FACTOR	TOTAL REVENUE FROM SIZE FACTOR *	PER PUPIL REVENUE FROM SIZE FACTOR *
48	ELBERT	AGATE	80.1	2.2871	390,015	4,869
49	EL PASO	CALHAN	577.5	1.2186	477,572	827
50	EL PASO	HARRISON	10,236.3	1.0120	464,687	45
51	EL PASO	WIDEFIELD	8,231.0	1.0120	373,654	45
52	EL PASO	FOUNTAIN	4,455.0	1.0283	476,947	107
53	EL PASO	COLORADO SPRINGS	31,491.8	1.0319	3,800,358	121
54	EL PASO	CHEYENNE MOUNTAIN	3,879.0	1.0362	531,208	137
55	EL PASO	MANITOU SPRINGS	1,368.5	1.1076	557,049	407
56	EL PASO	ACADEMY	15,886.5	1.0120	721,184	45
57	EL PASO	ELLICOTT	829.0	1.1668	523,103	631
58	EL PASO	PEYTON	720.0	1.1892	515,335	716
59	EL PASO	HANOVER	213.0	1.7872	634,309	2,978
60	EL PASO	LEWIS-PALMER	4,165.0	1.0323	508,925	122
61	EL PASO	FALCON	5,334.0	1.0163	328,910	62
62	EL PASO	EDISON	67.0	2.3364	338,725	5,056
63	EL PASO	MIAMI-YODER	317.5	1.4805	577,130	1,818
64	FREMONT	CANON CITY	4,311.0	1.0303	494,148	115
65	FREMONT	FLORENCE	1,937.5	1.0770	564,376	291
66	FREMONT	COTOPAXI	334.0	1.4528	572,123	1,713
67	GARFIELD	ROARING FORK	4,806.5	1.0235	427,300	89
68	GARFIELD	RIFLE	3,473.0	1.0417	547,870	158
69	GARFIELD	PARACHUTE	726.0	1.1880	516,334	711
70	GILPIN	GILPIN	372.0	1.3890	547,430	1,472
71	GRAND	WEST GRAND	534.0	1.2276	459,780	861
72	GRAND	EAST GRAND	1,234.5	1.1148	536,129	434
73	GUNNISON	GUNNISON	1,649.0	1.0925	577,030	350
74	HINSDALE	HINSDALE	63.8	2.3484	325,444	5,101
75	HUERFANO	HUERFANO	861.9	1.1600	521,691	605
76	HUERFANO	LA VETA	304.8	1.5019	578,720	1,899
77	JACKSON	NORTH PARK	305.6	1.5005	578,620	1,893
78	JEFFERSON	JEFFERSON	86,151.5	1.0342	11,146,160	129
79	KIOWA	EADS	285.5	1.5343	577,069	2,021
80	KIOWA	PLAINVIEW	85.0	2.2687	407,957	4,799
81	KIT CARSON	ARRIBA-FLAGLER	232.0	1.7157	628,138	2,707
82	KIT CARSON	HI PLAINS	115.0	2.1558	502,825	4,372
83	KIT CARSON	STRATTON	286.7	1.5322	577,217	2,013
84	KIT CARSON	BETHUNE	168.5	1.9546	608,496	3,611
85	KIT CARSON	BURLINGTON	844.0	1.1637	522,670	619
86	LAKE	LAKE	1,213.3	1.1160	532,430	439
87	LA PLATA	DURANGO	4,733.0	1.0245	438,671	93
88	LA PLATA	BAYFIELD	1,053.0	1.1246	496,344	471
89	LA PLATA	IGNACIO	1,089.0	1.1227	505,486	464
90	LARIMER	POUDRE	22,735.0	1.0120	1,032,078	45
91	LARIMER	THOMPSON	13,934.0	1.0120	632,548	45
92	LARIMER	ESTES PRK	1,304.0	1.1111	548,060	420
93	LAS ANIMAS	TRINIDAD	1,525.5	1.0991	571,903	375
94	LAS ANIMAS	PRIMERO	187.9	1.6816	626,664	3,335
95	LAS ANIMAS	HOEHNE	334.0	1.4528	572,123	1,713
96	LAS ANIMAS	AGUILAR	167.8	1.9572	607,618	3,621
97	LAS ANIMAS	BRANSON	51.3	2.3954	270,802	5,279
98	LAS ANIMAS	KIM	79.0	2.2912	385,884	4,885

	COUNTY	DISTRICT	EST. OCT 99 FUNDED PUPIL COUNT	EST. FY99-00 SIZE FACTOR	TOTAL REVENUE FROM SIZE FACTOR *	PER PUPIL REVENUE FROM SIZE FACTOR *
99	LINCOLN	GENOA-HUGO	295 0	1 5183	578,415	1,961
100	LINCOLN	LIMON	628 5	1 2081	494,782	787
101	LINCOLN	KARVAL	88 4	2 2559	419,995	4,751
102	LOGAN	VALLEY	2,742 5	1 0517	536,381	196
103	LOGAN	FRENCHMAN	221 5	1 7552	632,808	2,857
104	LOGAN	BUFFALO	270 2	1 5720	584,679	2,164
105	LOGAN	PLATEAU	155 5	2 0035	590,315	3,796
106	MESA	DEBEQUE	170 0	1 9489	610,247	3,590
107	MESA	PLATEAU	551 4	1 2240	467,252	847
108	MESA	MESA VALLEY	18,399 0	1 0120	835,241	45
109	MINERAL	CREEDE	156 0	2 0016	591,092	3,789
110	MOFFAT	MOFFAT	2,622 3	1 0533	528,745	202
111	MONTEZUMA	MONTEZUMA	3,497 0	1 0414	547,687	157
112	MONTEZUMA	DOLORES	637 8	1 2062	497,519	780
113	MONTEZUMA	MANCOS	534 5	1 2275	460,008	861
114	MONTROSE	MONTROSE	5,125 0	1 0192	372,247	73
115	MONTROSE	WEST END	501 0	1 2344	444,254	887
116	MORGAN	BRUSH	1,500 5	1 1005	570,477	380
117	MORGAN	FT MORGAN	3,035 5	1 0477	547,753	180
118	MORGAN	WELDON	149 0	2 0279	579,393	3,889
119	MORGAN	WIGGINS	565 8	1 2210	473,033	836
120	OTERO	EAST OTERO	1,935 3	1 0771	564,468	292
121	OTERO	ROCKY FORD	1,083 9	1 1229	503,938	465
122	OTERO	MANZANOLA	276 3	1 5497	574,570	2,080
123	OTERO	FOWLER	380 9	1 3741	539,057	1,415
124	OTERO	CHERAW	226 3	1 7372	631,112	2,789
125	OTERO	SWINK	344 5	1 4352	567,172	1,646
126	OURAY	OURAY	247 0	1 6593	616,051	2,494
127	OURAY	RIDGWAY	283 3	1 5379	576,480	2,035
128	PARK	PLATTE CANYON	1,525 4	1 0992	572,442	375
129	PARK	PARK	578 5	1 2184	477,961	826
130	PHILLIPS	HOLYOKE	643 5	1 2050	499,044	776
131	PHILLIPS	HAXTUN	305 8	1 5002	578,652	1,892
132	PITKIN	ASPEN	1,274 5	1 1127	543,376	426
133	PROWERS	GRANADA	297 5	1 5141	578,590	1,945
134	PROWERS	LAMAR	1,936 3	1 0770	564,027	291
135	PROWERS	HOLLY	348 5	1 4285	564,924	1,621
136	PROWERS	WILEY	344 0	1 4360	567,389	1,649
137	PUEBLO	PUEBLO CITY	17,118 5	1 0120	777,111	45
138	PUEBLO	PUEBLO RURAL	6,627 0	1 0120	300,839	45
139	RIO BLANCO	MEEKER	716 1	1 1900	514,711	719
140	RIO BLANCO	RANGELY	728 6	1 1875	516,805	709
141	RIO GRANDE	DEL NORTE	767 6	1 1794	520,947	679
142	RIO GRANDE	MONTE VISTA	1,426 0	1 1045	563,731	395
143	RIO GRANDE	SARGENT	419 9	1 3086	490,205	1,167
144	ROUTT	HAYDEN	528 5	1 2287	457,243	865
145	ROUTT	STEAMBOAT SPRINGS	1,911 5	1 0784	566,926	297
146	ROUTT	SOUTH ROUTT	460 0	1 2428	422,516	919
147	SAGUACHE	MTN VALLEY	187 5	1 8831	626,394	3,341
148	SAGUACHE	MOFFAT	196 0	1 9640	714,775	3,647
149	SAGUACHE	CENTER	702 9	1 1928	512,669	729

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	COUNTY	DISTRICT	EST. OCT 99 FUNDED PUPIL COUNT	EST. FY99-00 SIZE FACTOR	TOTAL REVENUE FROM SIZE FACTOR *	PER PUPIL REVENUE FROM SIZE FACTOR *
150	SAN JUAN	SILVERTON	93 1	2 2382	436,091	4,684
151	SAN MIGUEL	TELLURIDE	498 0	1 2350	442,724	889
152	SAN MIGUEL	NORWOOD	313 3	1 4876	577,910	1,845
153	SEDGWICK	JULESBURG	319 0	1 4780	576,839	1,808
154	SEDGWICK	PLATTE VLY	139 3	2 0644	560,909	4,027
155	SUMMIT	SUMMIT	2,491 0	1 0551	519,232	208
156	TELLER	CRIPPLE CREEK	597 5	1 2145	484,843	811
157	TELLER	WOODLAND PARK	3,176 0	1 0457	549,077	173
158	WASHINGTON	AKRON	477 4	1 2392	431,996	905
159	WASHINGTON	ARICKAREE	128 5	2 1050	537,158	4,180
160	WASHINGTON	OTIS	194 3	1 8575	630,294	3,244
161	WASHINGTON	LONE STAR	92 0	2 2423	432,365	4,700
162	WASHINGTON	WOODLIN	128 0	2 1069	535,988	4,187
163	WELD	GILCREST	1,899 9	1 0790	567,798	299
164	WELD	EATON	1,327 3	1 1098	551,325	415
165	WELD	KEENESBURG	1,505 0	1 1002	570,480	379
166	WELD	WINDSOR	2,404 0	1 0563	512,011	213
167	WELD	JOHNSTOWN	1,476 0	1 1018	568,421	385
168	WELD	GREELEY	14,370 0	1 0120	652,341	45
169	WELD	PLATTE VLY	1,108 5	1 1216	509,924	460
170	WELD	FORT LUPTON	2,477 0	1 0553	518,188	209
171	WELD	AULT-HGHLND	893 0	1 1536	518,894	581
172	WELD	BRIGGSDALE	131 5	2 0937	544,077	4,137
173	WELD	PRAIRIE	120 6	2 1348	517,730	4,293
174	WELD	GROVER	137 9	2 0697	558,036	4,047
175	YUMA	WEST YUMA	1,018 0	1 1279	492,555	484
176	YUMA	EAST YUMA	960 5	1 1397	507,610	528
	**STATE	TOTAL**	683,040 5	1 0492	$127,185,089	$186

* The funding shown is calculated using the FY 1998-99 base per pupil funding amount ($3,783)

Minimum per pupil funding is often confused with the size factor. One aspect of the school finance act that sometimes creates confusion is the relationship between the minimum size factor and "minimum per pupil funding" districts. Prior to FY 1998-99, the minimum size factor was 1.0000 and districts with a size factor of 1.0000 received no additional funding from the size factor. Meanwhile, a different provision of the act guaranteed that some districts would receive a minimum per pupil amount that was greater than what the district would have received from its factors. These eight districts are often called "minimum per pupil funding" districts.

Some districts with a very low size factor also receive minimum per pupil funding, so many people confuse the issues as the same. In reality, factors other than the size factor may contribute to a district's eligibility under the minimum per pupil funding provisions in the act. Table 4, below, provides data on minimum per pupil funding districts and districts receiving the minimum size factor in FY 1998-99.

TABLE 4: FY 1998-99 Data for Selected Districts[7]

County	District	Minimum Per Pupil Funding District?	FY 1998-99 Size Factor	FY 1998-99 Cost-of-Living Factor	FY 1998-99 At-Risk Factor	Est. FY 1998-99 Per Pupil Funding
Adams	Commerce City	no	1.0081	1.207	20.16%	$4,887
Adams	Westminster	no	1.0081	1.207	14.31%	$4,700
Arapahoe	Littleton	no	1.0081	1.227	11.50%	$4,632
Boulder	St. Vrain	no	1.0081	1.227	11.50%	$4,689
El Paso	Academy	yes	1.0081	1.208	11.50%	$4,572
El Paso	Harrison	no	1.0081	1.198	16.21%	$4,718
El Paso	Widefield	yes	1.0081	1.168	11.50%	$4,574
Fremont	Canon City	yes	1.0267	1.146	12.23%	$4,556
Larimer	Poudre	yes	1.0081	1.178	11.50%	$4,563
Larimer	Thompson	yes	1.0081	1.178	11.50%	$4,562
Mesa	Mesa Valley	yes	1.0081	1.137	13.30%	$4,566
Moffat	Moffat	yes	1.0519	1.127	11.50%	$4,569
Pueblo	Pueblo City	no	1.0081	1.167	19.29%	$4,716
Pueblo	Pueblo Rural	yes	1.0081	1.157	11.50%	$4,516
Weld	Greeley	no	1.0081	1.177	16.35%	$4,650

7. These selected districts include those funded under the minimum per pupil funding provisions and those that receive the minimum size factor in FY 1999-00. The data are estimated based on House Bill 98-1234, as adopted by the General Assembly.

APPENDIX 1 - CALCULATING THE SIZE FACTOR

Table 5, below, provides the formula for calculating a school district's size factor for FY 1999-00 under the provisions of House Bill 98-1234. The statutory formula is found in Section 22-54-104 (5) (b), C.R.S.

TABLE 5. Statutory Formula for Calculating School District Size Factors in FY 1999-00

If a district's funded pupil count is:	The district's size factor shall be:
Less than 276	1.5502 + (0.00376159 x the difference between the funded pupil count and 276)
276 or more but less than 459	1.2430 + (0.00167869 x the difference between the funded pupil count and 459)
459 or more but less than 1,027	1.1260 + (0.00020599 x the difference between the funded pupil count and 1,027)
1,027 or more but less than 2,293	1.0578 + (0.00005387 x the difference between the funded pupil count and 2,293)
2,293 or more but less than 5,650	1.0120 + (0.00001642 x the difference between the funded pupil count and 5,650)
5,650 or more but less than 25,546	1.0120
25,546 or more but less than 32,193	1.0120 + (0.00000334 x the difference between the funded pupil count and 25,546)
32,193 or more	1.0342

APPENDIX 2 - LOWESS
A Statistical Method to Determine Lines of Best Fit

LOWESS[8] is a statistical smoothing method that employs weighted least squares to fit a curve to a scatter plot. To start, an x-value on the scatter plot is chosen as the point of interest to which a y-value will be matched for the LOWESS curve. Next, the user establishes a percentage of the total points on the plot that will be used to create a range around the point of interest (i.e., if there are 40 points on the scatter plot and the user chooses 50 percent, then the 20 nearest points, as measured by their distance along the x-axis from the point of interest, would be used). Weights are then assigned to the points being used, with the nearest point to the x-value of interest receiving the highest weight and the furthest point receiving the lowest weight. A line is then fit by weighted least squares to the points being used. The y-value for the point on the fitted line that corresponds to the chosen x-value is then used as the y-value for the LOWESS curve at that x-value. At this time, one x,y-point on the LOWESS curve has been found. A new x-value is chosen, and the process is repeated until the entire LOWESS curve has been created.

Example:

Chart 3, on the next page, illustrates the steps used to find one x,y point for the fitted LOWESS curve. There are 20 points in the scatter plot and 50 percent of the points will be used at any one time. In step 1, the point x_6 has been chosen as the point of interest. The ten closest points (50 percent of 20) to x_6 along the x-axis are isolated as the points that will be used to draw the fitted line. Step 2 assigns a weight function to the points so that the points closest to x_6 receive the most weight and those points outside of the range receive no weight. The weight given to a point is the height of the curve at x_i in the lower left panel. Most importantly, the point at x_6 must have the largest weight, the weight function must decrease smoothly as x values are further away from x_6, the weight function must be symmetrical around x_6, and the weight function must decline to zero as x reaches the 50 percent boundary.

The formula used to find the weight t_i for the specific point (x_k,y_k) when computing a smoothed value at x_i is:

$$t_i(x_k) = T((x_i - x_k) / d_i)$$

Where:

x_i = the x-value that has been chosen as the point of interest.

d_i = the distance from x_i to its qth nearest neighbor along the x-axis. Where q is f_n rounded to the nearest integer and f is approximately the fraction of points to be used in the computation of the fitted value (50 percent in this case).

(x_k,y_k) = the coordinates of the point which is being weighted.

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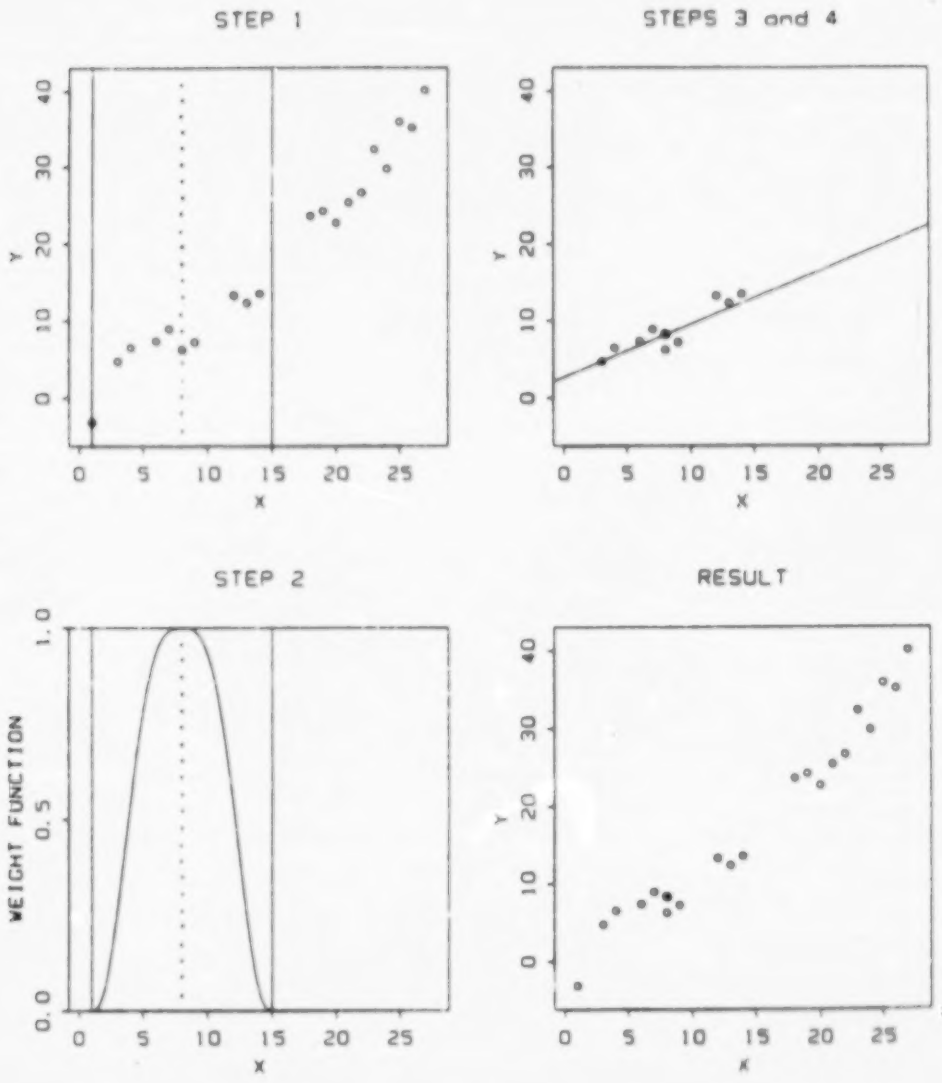
8. Chambers, J.M., W.S. Cleveland, B. Kleiner, and P.A. Tukey. *Graphic Methods for Data Analysis*, Belmont, California, Wadsworth International Group: Boston, Duxburg Press, 1983.

And, where the functional form of T is

$T(u) = (1 - |u|^3)^3$ for $|u| < 1$ and $T(u) = 0$ otherwise (the tricube weight function)

After the weights are assigned, a line is fit to the points on the scatter plot that have been isolated (50 percent of the values closest to x_i). The fitted line describes in a linear way how y depends on x within the interval. Steps 3 and 4 show the points within the 50 percent interval along with the fitted line. The fitted value for the LOWESS curve is defined to be the value of the fitted line at $x = x_6$. This point has been added to the scatter plot and is the solid point on the line. The process is repeated for every x value until all of the points for the LOWESS curve have been found.

CHART 3. Computing a Smoothed Value at x_6 Using Neighborhood Weights (LOWESS)



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